Optimal crop distribution in Vojvodina*

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Abstract

In this paper problem of optimal crop distribution is presented. Mathematical tools, as linear, nonlinear optimization, dynamic and stochastic programming are used.

1 Introduction

Let us consider production of cereals (wheat and maize) and production of industrial crops (sugar beet, sunflower and soybean) on farm in Vojvodina. Vojvodina is typicaly agricultural region, but because of bad financial situation, farmers are strongly risk-averse. Farmers, agronomists, and other agricultural specialists make a lot of decisions. On the one hand, farmers want maximization of the total gross margin, minimization of the total risk and minimization of the total labour. On the other hand, there are a lot of constraints (rotational constraint, policy constraint, market constraint).

This is a real-world problem and in global level is of big importance for every state. It is understandable that state should have agricultural policy and in interest of both stimulate farmers to produce certain crops in certain volume. Farmer's decision about crops that he will sow depends of same predictions about yields and expected prices, e.g. weather conditions, market requirement, stocks. Scientists should minimize risk for farmers.

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Overview of the paper: In Section 2, we give a description of the problem we are interested in. In Section 3 we describe mathematicaly our model. In Section 4 we give example and apply our model in order to get optimal crop distribution, and in Section 5 we draw final conclusions and discribe what can be done in future.

2 Problem Description

We consider the optimal crop distribution subject to maximization of the total gross margin and minimization of total risk. We are interested in distribution of five crops: wheat, maize, sugar beet, sunflower and soybean.

Optimal crop distribution can be considered as problem of optimization with constraints. In our problem we consider land constraint, budget limit and crop rotation constraint.

Farmer has limited size of land, let us assume N hectares.

Farmer has limited budget, so his costs cannot be exceeded this budget limit. Let us denote this budget limit with b. In real world, farmer can pay in seeds instead of money. In this way he will recover or save certain ammount of money and relax the budget constraint. This phenomenon is called the *parity*.

Crop rotations can be important for pest and disease control and for maintaining soil fertility. Crop rotational constraints are:

- In the same parcel sugar beet can be seeded once in five years
- If soybean was planted in some plot then sunflower cannot be planted in four years at that parcel, vice versa
- None of the crops can be planted two years in the same parcel.

There are two possible ways of implementing these crop rotation. One approach is to plant the entire farm to a single crop each year. In this case the same crop will only be grown again when its turn in the sequence arrives. This approach is hardly ever used in practice, because special machines and equippment are required for one type of crop. This approach is also a risky approach because the farmer is dependent on yield just on one type of crop. Second approach is to divide the land to several smaller parts of land (plots) and to treat those parts of lands as a whole. During one rotation period just one type of crop can be planted in one parcel. This assumption has large influence on solving problem of optimal crop distribution: one should consider this problem of optimization as disreet model. Output of model is optimal crop distribution on each parcel, i.e. we get answer on which parcel should we plant which crop. Also, the nature of these constraints require taking care for several years in past. This can be solved with dynamic programming.

Goal of this model is not just maximization of total gross margin, but minimisation of risk. Farmers face a variety of price, yield, and resource risks which make their incomes unstable from year to year. In many cases, farmers are also confronted by the risk of catastrophe. Numerous empirical studies have demonstrated that farmers typically behave in risk-averse ways (e.g. Binswanger 1980 and Dillon and Scandizzo 1978). As such, farmers often prefer farm plans that provide a satisfactory level of security even if this means sacrificing income on average. In a risky world a farm plan no longer has known income each year. Rather there are many possible income outcomes and, in the mathematical context, the actual outcome each year depends on the realized values of previous year. There are two types of risk: risk and uncertainty. If the probability distribution of outcome are known, we are talking about risk. If we don't know probability distribution of outcome, we are talking about uncertainty. In either case one can only form estimates of the possible income outputs.

In problem of optimal crop distribution there are variety of risks. There are risks as output price risk, yield risk, input price incertainty, government policy uncertainty, weather uncertainty, desease uncertainty, etc. In optimal crop distribution model we shall deal with yield, other risks will be neglected.

There are several approaches for modeling under risk. One of these approaches is the *Mean-Variance Approach.* This approach results from *expected utility theory* developed by *Von-Neumann and Morgenstern* in *The Theory of Games and Economic Behavior* in 1944. The Mean-Variance approach was introduced in works by *Tobin (1959)* and by *Markovitz (1959).* This approach assumes that a farmer's preferences among alternative farm plans are based on expected income and associated income variance. Main idea of this approach is next: it will return crop distribution which gives farmer maximum income and minimum variance, i.e. risk. It uses utility function which is function of distribution's mean and variance. Certain restrictive conditions on the utility function and the distribution of the random outcome variable are required in order to be able to express expected utility as a function of the mean and variance. These are:

- the utility function must be quadratic or exponential in form,
- the outcome variable should be normally distributed.

Unfortunately this model is objectionable on three grounds:

- Quadratic utility function implies increasing absolute risk-aversion, which is violation of expected utility theory,
- Exponential utility implies constant absolute risk-aversion, which is also violation of expected utility theory, and
- Normality of the outcome variable may be unreasonable.

Despite these shortcomings, the linear mean variance approach is popular since it results in models useful for dynamic programming.

The mean-variance approach can be very useful in modeling farmer's land allocation problem. Using matrix notation the farmer's problem can be stated as:

$$\max Y^T X - rX^T \Sigma X - T^T X,$$

where:

- Y is average yield vector,
- T is variable cost vector,
- Σ is variance-covariance matrix of yield per ha, and

- r is measure of risk. In order to make this model as realistic as possible, one should invest and make good approximation of risk coefficient r. It can been done with statistical calculations using data as yield, price fluctuation from year to year.

As input of model, we used next data:

- Yields (t/ha) in period of last five years (2000-2004)

	2000	*2001*	*2002*	*2003*	*2004*	*Mean*	*Variance*
Wheat	3.4	4.1	3.4	2.3	4.7	3.58	0.6456
Maize	2.9	5.6	5.0	3.4	5.9	4.56	1.4344
Soybean	1.2	2.4	2.5	1.7	2.7	2.1	0.316
Sunflower	1.5	2.0	1.9	1.8	2.3	1.9	0.068
Sugar beat	24.7	42.7	41.0	27.4	46.7	36.5	76.956

- Variance-covariance table of yields (t/ha) in period of last five years (2000-2004)

	Wheat	*Maize*	*Soybean*	*Sunflower*	*Sugar beet*
Wheat	0.6456	0.7492	0.286	0.14	5.522
Maize	0.7492	1.4344	0.65	0.284	10.448
Soybean	0.286	0.65	0.316	0.134	4.808
Sunflower	0.14	0.284	0.134	0.068	2.066
Sugar beat	5.522	10.448	4.808	2.066	76.956

- Costs (din/ha) - for farmer without his own mechanization

Wheat	30586.40
Maize	27497.11
Soybean	23010.20
Sunflower	26841.32
Sugar beet	58769.11

- Parities (t/ha) table

Material & labour	*Wheat*	*Maize*	*Sunflower*	*Soybean*	*Sugarbeet*
Seed	0.7	0.40602	0.173	0.22448	4.512
NPK f.	1	1.149214286	0.2325	0.465	5.58
Mineral f.	0.4	0.486871429	0	0	1.914
Combain h.	0.36	0.571428571	0.2	0.235714286	3.825
Herbicide	0	0.258214286	0.0995	0.0909	1.01988
Total	2.46	2.871748571	0.705	1.016094286	16.85088

- Price for crops (din/t) in last five years - 2000-2004

	2000	*2001*	*2002*	*2003*	*2004*	*Mean*
Wheat	4000	7500	7000	8000	6500	6600
Maize	4000	4500	5000	7000	6500	5400
Soybean	9000	13000	13000	13000	13000	12200
Sunflower	6000	12000	12000	12000	12000	10800
Sugar beat	1670	1860	1920	1920	2000	1874

3 Mathematical Model

In order to present our model, let us denote with:

- Farmer has N hectares of land, and m parcels.
- Ammount of avalaible budget is b.
- Selling price $c_i,\ i = 1,\dots,5$, and yields $y_i,\ i = 1,\dots,5$, are unknown - they are predicted in terms of mathematical expectation of data given from last five years. These predictions are given in Yield and Price tables as column "Mean". We consider wheat as 1-st crop, maize as 2-nd crop, soybean as 3-rd crop, sunflower as 4-th crop and sugar beet as 5-th crop.
- With $x_i,\ i = 1,\dots,5$, is denoted the number of hectares planted with i-th crop.
- With $X_{i,j},\ j = 1,\dots,m,\ i = 1,\dots,5$, is denoted the size in hectares of j-th parcel planted with i-th crop. It actualy represents the unique identifeier of parcel. Output of model is the answer: on which plot should we plant which seed. Because of this nature of model, we call this model discrete model. One can see that $x_i = \sum_{j=1}^{m} X_{i,j},\ i = 1\dots,5$ and $\sum_{i=1}^{5}\sum_{j=1}^{m} X_{i,j} = N$.
- With $t_i,\ i = 1,\dots,5$, are denoted costs of i-th crop, $i = 1,\dots,5$. These costs are under assumption that farmer doesn't has own mechanization, so he rents it. In order to make optimal crop distribution model as realistic as can, one could take into consideration parities. If we denote with p_i the ammount of seeds in t which can be given instead of money for, $i = 1,\dots,5$, costs can be written as follows:

  $$t_i^p = t_i - \text{ (costs of chosen action)}$$

- Rotation constraints are denoted with rot, and they can be applied to parcels $X_{i,j},\ j = 1,\dots,m,\ i = 1,\dots,5$.
- Let us denote with σ_i^2 the variance of i-th crop yield, $i = 1,\dots,5$. They are also given in table of yields, and one can calculate them with formula for variance.
- With σ_{ij} let us denote the covariance of i-th and j-th crop, $i,j = 1,\dots,5$. They are given in variance-covariance table and can be calculated with formula for covariance.

3.1 Objective - utility function

In its simplest form, objective function for maximizing total gross margin without parity and risk can be written as follows:

$$\sum_{i=1}^{5}(c_i y_i - t_i)x_i.$$

Considering parity, our function changes in this way:

$$\sum_{i=1}^{5}(c_i(y_i - p_i) - t_i^p)x_i.$$

Introducing yield risk in optimal crop distribution model, objective function can be written as follows:

$$\sum_{i=1}^{5}(c_i(y_i-p_i)-t_i^p)x_i-\frac{r}{2}[\sum_{i=1}^{5}x_i^2\sigma_i^2+\sum_{j\neq i}x_ix_j\sigma_{ij}]$$

By discretization of x_i, final form of objective function can be written as:

$$\sum_{i=1}^{5}[(c_i(y_i-p_i)-t_i^p)\sum_{j=1}^{m}X_{i,j}]- \tag{1}$$
$$-\frac{r}{2}[\sum_{i=1}^{5}(\sum_{j=1}^{m}X_{i,j})^2\sigma_i^2+\sum_{l\neq i}(\sum_{j=1}^{m}X_{i,j})(\sum_{j=1}^{m}X_{l,j})\sigma_{ij}]$$

3.2 Constraints of model

Constraints for optimal crop distribution problem can be written as:

$$\sum_{i=1}^{5}x_i=\sum_{i=1}^{5}\sum_{j=1}^{m}X_{i,j} \quad \leq \quad N \tag{2}$$

$$\sum_{i=1}^{5}t_i^p x_i=\sum_{i=1}^{5}t_i^p\sum_{j=1}^{m}X_{i,j} \quad \leq \quad b \tag{3}$$

$$\sum_{j=1}^{m}rot(X_{i,j}) \quad \leq \quad N \tag{4}$$

$$x_i=\sum_{j=1}^{m}X_{i,j} \quad \geq \quad 0,\ i=1,\ldots,5, \tag{5}$$

$$y_i-p_i \quad \geq \quad 0,\ i=1,\ldots,5, \tag{6}$$

where constraints can be eplained as follows: (2) stands for land constraint, (3) stands for budget constraint, (5) stands for constaint for nonnegativity, (6) stands for condition that yield of i-th crop should be as least as parity for the same crop and (4) stands for rotation constraints.

3.3 Optimal crop distribution model - problem of quadratic programming with constraints

Final model of optimal crop distribution can be restated as follows:

$$\sum_{i=1}^{5}[(c_i(y_i-p_i)-t_i^p)\sum_{j=1}^{m}X_{i,j}]-$$
$$-\frac{r}{2}[\sum_{i=1}^{5}(\sum_{j=1}^{m}X_{i,j})^2\sigma_i^2+\sum_{l\neq i}(\sum_{j=1}^{m}X_{i,j})(\sum_{j=1}^{m}X_{l,j})\sigma_{ij}]\rightarrow \max$$

subject to following constraints:

$$\begin{aligned}
\sum_{i=1}^{5}\sum_{j=1}^{m} X_{i,j} &\leq N \\
\sum_{i=1}^{5} t_i^p \sum_{j=1}^{m} X_{i,j} &\leq b \\
\sum_{j=1}^{m} rot(X_{i,j}) &\leq N \\
x_i = \sum_{j=1}^{m} X_{i,j} &\geq 0,\ i = 1,\ldots,5, \\
y_i - p_i &\geq 0,\ i = 1,\ldots,5,
\end{aligned}$$

This model can be considered as quadratic program with constraints. Let us consider the simplified matrix form of this problem:

$$y^T x - \frac{r}{2} x^T \Sigma x \to \max$$

subject to constraints:

$$Ax \leq b,$$

where y is average yield vector, Σ is variance-covariance matrix, A is constraint matrix and b is vector.
This problem can be solved by introducing Lagrangian:

$$L = \max_x y^T x - \frac{r}{2} x^T \Sigma x + \lambda(b - Ax).$$

The first order condition can be written as system, of equation:

$$\begin{aligned}
\frac{\partial L}{\partial x} &= y^T - r\Sigma x - \lambda A = 0 \\
\frac{\partial L}{\partial \lambda} &= b - Ax = 0
\end{aligned}$$

At the optimal solution, $\lambda = \bar{y} - \bar{MV}$, where $\bar{y} = \frac{1}{5}\sum_{i=1}^{5} y_i$ and $\bar{MV} = \frac{1}{5}\sum_{i=1}^{5} MV_i$. MV_i is risk cost. The first order condition $\frac{\partial L}{\partial x} = 0$ can be rewritten as:

$$y_i - \bar{y} - (MV_i - \bar{MV}) = 0.$$

From last equation one can see that a crop will be planted if it is either more profitable or less risky than average. Riskiness may not reflect less variability, but rather, negative correlation of profit with other crops.

Rotation constraints involve several years, so every year depends on previous year, and one can use techniques for dynamic programming with two stages in solving this problem. In first stage, one can use data for years 2000, 2001, 2002, 2003, 2004, and in second stage one should solve optimal crop distribution with stochastic yields. In order to implement rotation constraint (4) with dynamic programming, one can use the next algorythm:

Let us construct a 3 dimensional matrix with elements 0 and 1. The size of the matrix will be $g * m * v$ where g stands for the number of years, m for the number of parcels and v for the number of crops. In our problem, $v = 5$.

For $i = 1, \ldots, v$, $j = 1, \ldots, g$ and for $k = 1, \ldots, m$, the entries of the matrix are the following:

$$a_{ijk} = \begin{cases} 1 & \text{, if } i\text{th crop is planted in the } j\text{th year on the } k \text{ th parcel} \\ 0 & \text{, else} \end{cases}$$

Rotation constraints for matrix can be written in next way:

1. If $a_{ijk} = 1$ then $a_{i(j+1)k} = 0$
2. If any of $a_{3(j-3)k}, a_{3(j-2)k}, a_{3(j-1)k}$ is 1, then $a_{4jk} = 0$
3. If any of $a_{4(j-3)k}, a_{4(j-2)k}, a_{4(j-1)k}$ is 1, then $a_{3jk} = 0$
4. If any of $a_{5(j-1)k}, a_{5(j-2)k}, a_{5(j-3)k}, a_{5(j-4)k}$ is 1, then $a_{5jk} = 0$
5. $\sum_{i=1}^{5} a_{ijk} = 1$

The first constraint means that no crop can be planted on the same parcel in two following years, the second and the third means that sunflower cannot be planted on plots on which we planted soybean the previous four years and vice versa, and the last constraint means that once sugar beet planted on plot, we cannot sow it on the same parcel for four years. Additionaly we extended these constraints with constraint which avvoids overlapping. Rotation constraints take into account years, parcels and crops. With this overlapping constraint, one could avoid that the same crop will be planted on the same parcell in different years which are unaccaptable according to rotation constaints.

Algorythm finds the matrix with the largest profit under the matrices satisfying all constraints.

Inputs of algorythm are: yields, incomes, costs and distributions from the previous years (for our experiments we used random matrices satisfying the previous constraints). The output of our program is the optimal distribution, and the maximum total gross margin.

4 Application of optimal crop distribution model to farm in Vojvodina

In region of Vojvodina most of farmers have land of size 15-50 ha, fewer have 50-200 ha and some agricultural cooperatives have over 200 ha.

Let us observ farmer with 100 ha of land and suppose that he has six plots of size $(15, 20, 10, 30, 7, 18)$ ha respectively. He must decide which crop will he plant on which plot in order to maximize total gross margin. Crops available are wheat, maize, sugar beet, sunflower and soybean. As the other farmers in region of Vojvodina, this farmer is also strongly risk-averse, so his utility function is concave.

In order to solve his problem, farmer can use model of optimal crop distribution described in this paper. Tables in chapter (2) can be taken as input, and farmer's problem can be written as stochastic optimization problem with objective function (1) and constraints (2), (3), (4), (5) and (6). To solve this problem, farmer takes year 2001 as starting year. In that year, crop distribution on his plots was following: (soybean, sunflower, wheat, wheat, wheat, wheat).

Shortcoming of optimal crop distribution model is that cannot give exact solution: in this paper risk coeficinet r is not approximated. Sollution will depend on r. This model can offer several sollutions for different r, and farmer can choose the distribution as sollution according to his risk-aversion level. If level of risk-averseness is higher, r is bigger and vice versa.

In order to help farmer in decission making, model can provide table with optimal crop distributons with different risk-aversion coefficients:

r	*1. plot*	*2. plot*	*3. plot*	*4. plot*	*5. plot*	*6. plot*	*Max. prof.*
1	sunflower	soybean	sugar beet	soybean	soybean	sugar beet	147198
4	sunflower	soybean	sugar beet	soybean	soybean	sugar beet	119527
8	sunflower	soybean	sugar beet	soybean	soybean	sugar beet	82632.10
16	sunflower	soybean	sugar beet	soybean	soybean	sugar beet	8842.65
17	maize	wheat	wheat	wheat	wheat	wheat	0
25	maize	wheat	wheat	wheat	wheat	wheat	0
100	maize	wheat	wheat	wheat	wheat	wheat	0

From this table one can conclude that sugar beet, sunflower and soybean are much more risky crops as wheat and maize. More risk-averse farmers should plant wheat and maize, and farmers with lower level of risk-aveseness should chose other three crops. This table also confirms that crop will be planted if it is either more profitable or less risky than average as stated in chapter (3.2).

5 Conclusions and furhter work

Vojvodina is agricultural region in Serbia. It has importan role in supplying Serbia with agricultural products. Government of Serbia should take into consideration the importance of agriculture in this region and apply correct and appropriate agricultural policy. Different mathematical models can help in creating better agricultural policy. Optimal crop distribution model can be one of these models. It also can help individual farmers to improve their agricultural production.

Model presented in this paper can be used in individual farms. In future work it can be improved for larger regions, as whole Vojvodina. Also, agricultural farm models are not realistic, if they don't take into account risk. This model can be improved by collecting more data about risk factors, and by applying on them statistic calculations in order to get more realistic scenarios. Very important taks is good approximation of risk coefficient r. Also, with different kind of surveys one can get more realistic utility function for risk measurement, and then for risk minimization in models. Also one can apply different risk minimization techniques in order to see which technique is more appropriate in given circumstances in Vojvodina.

Improving optimal crop distribution model for several years strategy could be also usefull for farmers. Such model should give optimal crop distribution for sevaral years in order to maximize total gross margin for period of more than one year.

References

[1] Peter B. R. Hazell, Roger D. Norton: Mathematical programming for economic analysis in agriculture, Macmillan Publishing Company, New York

[2] David Zilberman: Agricultural and environmental policies, Class notes from the ARE242 course, Spring 1998

[3] L. Júdez, J. M. de Miguel, J. Mas and R. Bru: Modeling crop regional production using positive mathematical programming, Mathematical and computer modeling

[4] M. A. H. Dempster: Stochastic Programming, Academic Press, 1980

[5] Ahmed Barkaoui, Jean-Pierre Butault: Cerals and oilseeds supply within the EU, under Agenda 2000: a positive mathematical programming application, Agricultural economics review, August 2000,Vol.1, No.2

[6] Markus Kempen, Thomas Heckelei, Wolfgang Britz: An econometric approach for spatial dissagregation of crop production in the EU

[7] Carl R. Dillon, Scott A. Sheare, Thomas Mueller: A mixed integer, nonlinear programming model of innovative variable rate planting date with polymer seed coatings, Presented at the America Agriculture Economics Association annual meeting, August 5-8, 2001, Chicago, Illinois

[8] K. Ververidis, N. McRoberts, A. Barnes: Evolutionary agricultural policy analysis, Land Economy and Environmental Research Group, Scottish Agricultural

[9] John R. Birge, François Louveaux: Introduction to stochastic programming, Springer-Verlag New York, Inc, 1997